1.1 Calculate VaR using a historical simulation approach.

- **Historical simulation approach (歷史模擬法)**

  (1) The simplest way to estimate VaR is by means of historical simulation (HS). The HS approach estimates VaR by means of orderedobservations. (將損失資料排序)

  (2) Suppose we have 1,000 observations and the VaR at the 95% confidence level. Since the confidence level implies a 5% tail, there are 50 observations in the tail, and we can compute the VaR to be the 51th highest loss observation.

Example1:ConceptionofVaRusingthehistoricalsimulationapproach

The VaR at a 95% confidence level is estimated to be 1.56 from a historical simulation of 1,000 observations. Which of the following statements is most likely true?

A. The historical simulation assumption of normal returns is correct.

B. The historical simulation assumption of lognormal returns is correct.

C. The historical distribution has fatter tails than a normal distribution.

D. The historical distribution has thinner tails than a normal distribution.

Ans: D

1.2 Calculate VaR using a parametric estimation approach assuming that the return distribution is either normal or lognormal.

In contrast to the historical simulation method, the parametric approach (e.g., the delta-normal approach) explicitly assumesadistribution for the underlying observations.

- **Parametric estimation approach for VaR**

  (1) Normal VaR

  Suppose the arithmeticreturns follow a normal distribution.

  $$r_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}} \sim N(\mu, \sigma^2)$$

  where P_t: asset price at the end of periods; D_t: interim payments

  $$\mathbf{VaR}(\alpha\%) = -(\mu + z_\alpha \sigma) \times P_{t-1}$$

Example2:CalculatenormalVaRwithparametricapproach

Suppose arithmetic returns over some period are distributed as normal with mean 0.1 and standard deviation 0.25, and we have a portfolio currently worth 1 million. Calculate VaR at both the 95% and 99% confidence levels.

Ans:

$$VaR(5\%) = -(0.1 - 1.645 \times 0.25) \times 1m = 1.331m$$
$$VaR(5\%) = -(0.1 - 2.33 \times 1.25) \times 1m = 1.4825m$$

Example3:CalculatenormalVaRwithparametricapproach

If profit/loss over some period is normally distributed with mean 10 and standard deviation 20, then calculate VaR at both the 95% and 99% confidence levels.

Ans:

$$VaR(5\%) = -(10 - 1.645 \times 25) = 22.9$$
$$VaR(5\%) = -(10 - 2.33 \times 25) = 22.6$$

(2) Lognormal VaR

Suppose the geometric returns follow a normal distribution and the asset price follows a lognormal distribution.

$$R_t = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right) \sim N(\mu, \sigma^2)$$

$$\mathbf{VaR(\alpha\%) = (1 - e^{\mu + z_\alpha \sigma}) \times P_{t-1}}$$

Example4:CalculatelognormalVaRwithparametricapproach

Suppose geometric returns over some period are distributed as normal with mean 0.05 and standard deviation 0.20, and we have a portfolio currently worth 1 million. Calculate VaR at both the 95% and 99% confidence levels.

Ans:

$$VaR(5\%) = (1 - e^{0.05 - 1.645 \times 0.20}) \times 1m = 1.244m$$
$$VaR(1\%) = (1 - e^{0.05 - 1.33 \times 0.20}) \times 1m = 0.340m$$

Example5:ComparenormalandlognormalVaRwithparametricapproach

Suppose we make the empirically not too unrealistic assumptions that the mean and volatility of annualized returns are 0.10 and 0.40, and we have a portfolio currently worth 1 million. Assume 250 trading days to a year.

(1) Calculate the <u>daily</u> normal VaR and the daily lognormal VaR at the 95% confidence level.

(2) Calculate the <u>annually</u> normal VaR and the daily lognormal VaR at the 95% confidence level.

Ans:

(1) daily mean $= \frac{0.1}{250} = 0.0004$; daily standard deviation $= \frac{0.4}{\sqrt{250}} = 1.0253$

daily normal VaR(5%) $= -(1.0004 - 1.645 \times 1.0253) \times 1m = 1.0412m$

daily lognormal VaR(5%) $= (1 - e^{0.0004-1.645\times0.0253}) \times 1m = 1.0404m$

(2) annually normal VaR(5%) $= -(0.1 - 1.645 \times 1.4) \times 1m = 1.558m$

annually lognormal VaR(5%) $= (1 - e^{0.1-1.645\times0.4}) \times 1m = 0.428m$

The answers illustrate that <u>normalandlognormalVaRsaremuchthesameifwearedealingwith</u> <u>short holdingperiodsandrealisticreturnparameters</u>.


1.3 Calculate the expected shortfall given P/L or return data.

Expected shortfall (ES) 預期短缺

Expected shortfall (ES) is the expected loss given that the portfolio return already lies below the pre-specified worst case quantile return (i.e., below the 5th percentile return). In other words, <u>expected shortfall is the mean percent loss among the returns falling below the q-quantile</u>. Expected shortfall is also known as **conditional VaR** or **expected tail loss (ETL)**.

$$\text{ES}_\alpha = \frac{1}{1-\alpha} \int_\alpha^1 q_p \, dp$$

The ES is the average of the worst $100(1-\sigma)\%$ of losses, let $p = 1 - \alpha$
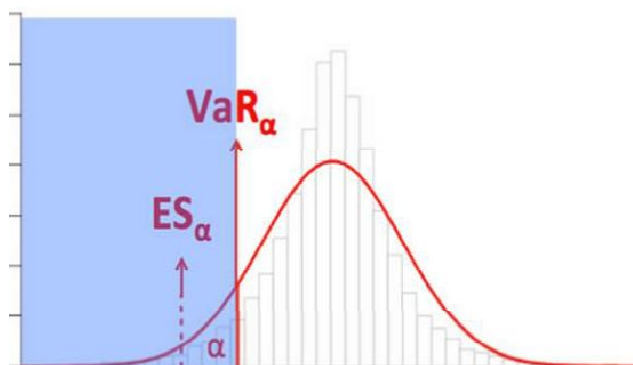
To illustrate the ES, suppose that we wish to estimate a 95% ES on the assumption that losses are normally distributed with mean 0 and standard deviation 1. In practice, we would use a high value of n and carry out the calculations on a spreadsheet or using appropriate software. However, to show the procedure manually, let us work with a very small n value of 10. This value gives us 9 (i.e., n-1) tail VaRs, or VaRs at confidence levels in excess of 95%. These VaRs are shown in the following table, and vary from 1.6954 (for the 95.5% VaR) to 2.5758 (for the 99.5% VaR). Our estimated ES is the average of these VaRs, which is 2.0250.

Confidence level	Tail VaR
95.5%	1.6954
96.0%	1.7507
96.5%	1.8119
97.0%	1.8808
97.5%	1.9600
98.0%	2.0537
98.5%	2.1701
99.0%	2.3263
99.5%	2.5738
Average of tail VaRs	ES = 2.0250

Of course, in using this method for practical purposes, we would want a value of n large enough to give accurate results. To give some idea of what this might be, the following table reports some alternative ES estimates obtained using this procedure with varying values of n. These results show that the estimated ES rises with n and gradually converges to the true value of 2.063. These results also show that our ES estimation procedure seems to be reasonably accurate even for quite small values of n. Any decent computer should therefore be able to produce accurate ES estimates quickly in real time.

Number of tail slices (n)	ES
10	2.0250
25	2.0433
50	2.0513
100	2.0562
250	2.0597
500	2.0610
1,000	2.0618
2,500	2.0623
5,000	2.0625
10,000	2.0626
True value	2.0630

Example6:Calculateexpectedshortfall(ES)

A market risk manager uses historical information on 200 days of profit/loss information to calculate a daily VaR at the 95th percentile, of USD 14 million. Loss observations beyond the 95th percentile are then used to estimate the conditional VaR. If the losses beyond the VaR level, in millions, are USD15, 17, 18, 20, 28, 30, 35, 40, 42, and 45, then what is the conditional VaR?

A. USD 20 million.
B. USD 25 million.
C. USD 29 million.
D. USD 32 million.

Ans: C

$$ES = \frac{15 + 17 + 18 + 20 + 28 + 30 + 35 + 40 + 42 + 45}{10} = 29m$$